Short Talk: Best Solution for FHERMA Lookup Table Challenge

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Workshop on Encrypted Computing & Applied Homomorphic Cryptography

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$\to$ Inspired by Iliashenko et al. [IZ21]

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Algorithm Fast Rotation and Add

procedure FastRotAdd(ct)
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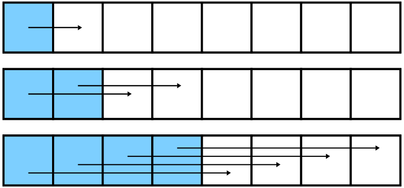

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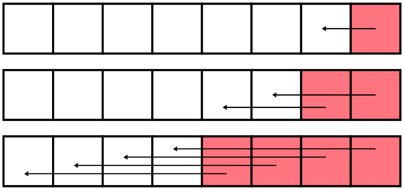

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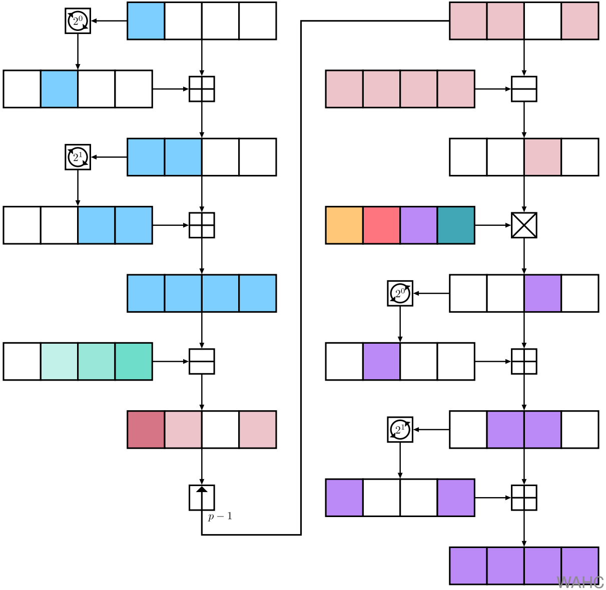

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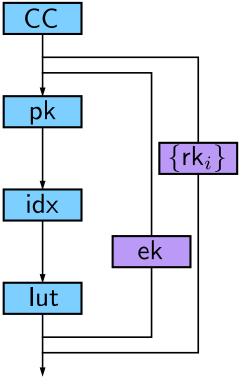
- Asynchronous key deserialization:



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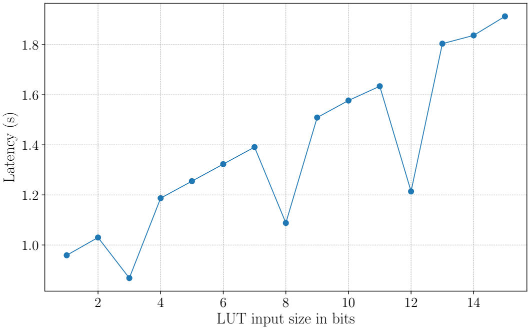

Figure: Performance of the proposed algorithm on a commodity laptop on OpenFHE v1.4.0

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$\log_2 p$	Latency (s)	Amz. time (ms)
10	46.17	0.70
12	52.46	0.80
14	98.94	1.51
16	112.17	1.71
20	1,014.8	7.74

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- Between 2-bit and 15-bit LUTs, could be further extended
- One of many components of the FHERMA cookbook (scan QR code for more details)
- Building blocks for easily creating FHE applications

Thank you !

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